

Intro to Fractions

Reading Fractions

Fractions are parts. We use them to write and work with amounts that are less than a whole number (one) but more than zero. The form of a fraction is one number over another, separated by a fraction (divide) line.

$$\text{i.e. } \frac{1}{2}, \frac{3}{4}, \text{ and } \frac{5}{9}$$

These are fractions. Each of the two numbers tells certain information about the fraction (partial number). The bottom number (denominator) tells how many parts the whole (one) was divided into. The top number (numerator) tells how many of the parts to count.

$$\frac{1}{2} \text{ says, "Count one of two equal parts."}$$

$$\frac{3}{4} \text{ says, "Count three of four equal parts."}$$

$$\frac{5}{9} \text{ says, "Count five of nine equal parts."}$$

Fractions can be used to stand for information about wholes and their parts:

EX. A class of 20 students had 6 people absent one day. 6 absentees are part of a whole class of 20 people. $\frac{6}{20}$ represents the fraction of people absent.

EX. A "Goodbar" candy breaks up into 16 small sections. If someone ate 5 of those sections, that person ate $\frac{5}{16}$ of the "Goodbar".

Exercise 1 Write fractions that tell the following information:

1. Count two of five equal parts
2. Count one of four equal parts
3. Count eleven of twelve equal parts
4. Count three of five equal parts
5. Count twenty of fifty equal parts
6. It's 25 miles to Gramma's. We have already driven 11 miles. What fraction of the way have we driven?
7. A pizza was cut into twelve slices. Seven were eaten. What fraction of the pizza was eaten?
8. There are 24 students in a class. 8 have passed the fractions test. What fraction of the students have passed fractions?

The Fraction Form of One

Because fractions show how many parts the whole has been divided into and how many of the parts to count, the form also hints at the number of parts needed to make up the whole thing. If the bottom number (denominator) is five, we need 5 parts to make a whole: $\frac{5}{5} = 1$. If the denominator is 18, we need 18 parts to make a whole of 18 parts: $\frac{18}{18} = 1$. Any fraction whose top and bottom numbers are the same is equal to 1.

Example: $\frac{2}{2} = 1$, $\frac{4}{4} = 1$, $\frac{100}{100} = 1$, $\frac{11}{11} = 1$, $\frac{6}{6} = 1$

Complementary Fractions

Fractions tell us how many parts are in a whole and how many parts to count. The form also tells us how many parts have not been counted (the complement). The complement completes the whole and gives opposite information that can be very useful.

$\frac{3}{4}$ says, "Count 3 of 4 equal parts." That means 1 of the 4 was not counted and is somehow different from the original 3.

$\frac{3}{4}$ implies another $\frac{1}{4}$ (its complement). Together, $\frac{3}{4}$ and $\frac{1}{4}$ make $\frac{4}{4}$, the whole thing.

$\frac{5}{8}$ says, "Count 5 of 8 equal parts." That means 3 of the 8 parts have not been counted, which implies another $\frac{3}{8}$, the complement. Together, $\frac{5}{8}$ and $\frac{3}{8}$ make $\frac{8}{8}$, which is equal to one.

Complementary Situations

It's 8 miles to town, We have driven 5 miles. That's $\frac{5}{8}$ of the way, but we still have 3 miles to go to get there or $\frac{3}{8}$ of the way.

$\frac{5}{8} + \frac{3}{8} = \frac{8}{8} = 1$ (1 is all the way to town).

A pizza was cut into 12 pieces. 7 were eaten $\frac{7}{12}$. That means there are 5 slices left or $\frac{5}{12}$ of the pizza. $\frac{7}{12} + \frac{5}{12} = \frac{12}{12} = 1$ (the whole pizza).

Mary had 10 dollars. She spent 5 dollars on gas, 1 dollar on parking, and 3 dollars on lunch. In fraction form, how much money does she have left?

Gas = $\frac{5}{10}$, parking = $\frac{1}{10}$, lunch = $\frac{3}{10}$

$\frac{5}{10} + \frac{1}{10} + \frac{3}{10} = \frac{9}{10}$; $\frac{1}{10}$ is the complement (the leftover money)

Altogether it totals $\frac{10}{10}$ or all of the money.

Exercise 2

Write the complements to answer the following questions:

1. A cake had 16 slices. 5 were eaten. What fraction of the cake was left?
2. There are 20 people in our class. 11 are women. What part of the class are men?
3. It is 25 miles to grandma's house. We have driven 11 miles already. What fraction of the way do we have left to go?
4. There are 36 cookies in the jar. 10 are Oreos. What fraction of the cookies are not Oreos?

Reducing Fractions

If I had 20 dollars and spent 10 dollars on a CD, it's easy to see I've spent half of my money. It must be that $\frac{10}{20} = \frac{1}{2}$. Whenever the number of the part (top) and the number of the whole (bottom) have the same relationship between them that a pair of smaller numbers have, you should always give the smaller pair answer. 2 is half of 4. 5 is half of 10. $\frac{1}{2}$ is the reduced form of $\frac{5}{10}$ and $\frac{2}{4}$ and $\frac{10}{20}$ and many other fractions.

A fraction should be reduced any time both the top and bottom number can be divided by the same smaller number. This way you can be sure the fraction is as simple as it can be.

$\frac{5}{10}$ both 5 and 10 can be divided by 5

$$\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

$\frac{1}{2}$ describes the same number relationship that $\frac{5}{10}$ did, but with smaller numbers. $\frac{1}{2}$ is the reduced form of $\frac{5}{10}$.

$\frac{6}{8}$ both 6 and 8 can be divided by 2.

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

$\frac{3}{4}$ is the reduced form of $\frac{6}{8}$.

When you divide both the top and bottom numbers of a fraction by the same number, you are dividing by a form of one so the value of the fraction doesn't change, only the size of the numbers used to express it.

$$\frac{12}{16} = \frac{12 \div 2}{16 \div 2} = \frac{6}{8}$$
 These numbers are smaller but they can go lower

because both 6 and 8 can be divided by 2 again. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

$$\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

$$\frac{27}{63} = \frac{27 \div 3}{63 \div 3} = \frac{9}{21} = \frac{9 \div 3}{21 \div 3} = \frac{3}{7} \quad \text{or} \quad \frac{27}{63} = \frac{27 \div 9}{63 \div 9} = \frac{3}{7}$$

Exercise 3. Try these. Keep dividing until you can't divide anymore.

1. $\frac{6}{8} =$

2. $\frac{12}{15} =$

3. $\frac{14}{18} =$

4. $\frac{8}{10} =$

5. $\frac{6}{12} =$

6. $\frac{16}{24} =$

Good knowledge of times tables will help you see the dividers you need to reduce fractions.

Here are some hints you can use that will help, too.

Hint 1

If the top and bottom numbers are both even, use $\frac{2}{2}$.

Hint 2

If the sum of the digits is divisible by 3 then use $\frac{3}{3}$.

$\frac{111}{231}$ looks impossible but note that 111 (1+1+1) adds up to three and 231 (2+3+1) adds up to 6. Both 3 and 6 divide by 3 and so will both these numbers: $\frac{111}{231} = \frac{111 \div 3}{231 \div 3} = \frac{37}{77}$

The new fraction doesn't look too simple, but it is smaller than when we first started.

Hint 3

If the 2 numbers of the fraction end in 0 and/or 5, you can divide by $\frac{5}{5}$.

$$\frac{45}{70} = \frac{45 \div 5}{70 \div 5} = \frac{9}{14}$$

Hint 4

If both numbers end in zeros, you can cancel the zeros in pairs, one from the top and one from the bottom. This is the same as dividing them by $\frac{10}{10}$ for each cancelled pair.

$$\frac{4000}{50000} = \frac{\cancel{4}\cancel{0}\cancel{0}\cancel{0}}{\cancel{5}\cancel{0}\cancel{0}\cancel{0}\cancel{0}} = \frac{4}{50} = \frac{4 \div 2}{50 \div 2} = \frac{2}{25}$$

Hint 5

If you have tried to cut the fraction by $\frac{2}{2}$, $\frac{3}{3}$, $\frac{5}{5}$ and gotten nowhere, you should try to see if the top number divides into the bottom one evenly. For $\frac{23}{69}$, none of the other hints help here, but $69 \div 23 = 3$. This means you can

$$\text{reduce by } \frac{23}{23}. \quad \frac{23}{69} = \frac{23 \div 23}{69 \div 23} = \frac{1}{3}$$

Exercise 4

Directions: Reduce these fractions to lowest terms

1. $\frac{14}{18} =$

2. $\frac{80}{100} =$

3. $\frac{18}{36} =$

4. $\frac{400}{5000} =$

5. $\frac{20}{25} =$

6. $\frac{27}{36} =$

7. $\frac{40}{45} =$

8. $\frac{63}{81} =$

9. $\frac{9}{12} =$

10. $\frac{60}{85} =$

11. $\frac{17}{51} =$

12. $\frac{50}{75} =$

Higher Equivalents

There are good reasons for knowing how to build fractions up to a larger form. It is exactly the opposite of what we do in reducing. If reducing is done by division, it makes sense that building up should be done by multiplication.

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

$$\frac{8}{9} = \frac{8 \times 6}{9 \times 6} = \frac{48}{54}$$

A fraction can be built up to an equivalent form by multiplying by any form of one, any number over itself.

$$\frac{2}{3} = \frac{2 \times 6}{3 \times 6} = \frac{12}{18}$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{2}{3} = \frac{2 \times 11}{3 \times 11} = \frac{22}{33}$$

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{2}{3} = \frac{12}{18} = \frac{8}{12} = \frac{22}{33} = \frac{6}{9} \quad \text{All are forms of } \frac{2}{3}; \text{ all will reduce to } \frac{2}{3}$$

Comparing Fractions

Sometimes it is necessary to compare the size of fractions to see which is larger or smaller, or if the two are equal. Sometimes several fractions must be placed in order of size. Unless fractions have the same bottom number (denominator) and thus parts of the same size, you can't know for certain which is larger or if they are equal.

Which is larger $\frac{2}{3}$ or $\frac{5}{6}$? Who knows? A ruler might help, but rulers aren't usually graduated in thirds or sixths. Did you notice that if 3 were doubled, it would be 6?

So build up $\frac{2}{3}$ by $\frac{2}{2}$; $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

Then it's easy to see that $\frac{5}{6}$ is larger because it counts more sixth parts than $\frac{4}{6}$, so $\frac{4}{6} < \frac{5}{6}$ means $\frac{2}{3} < \frac{5}{6}$

Which is larger $\frac{15}{16}$ or $\frac{3}{4}$?

Build up $\frac{3}{4}$ by $\frac{4}{4}$. $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$. $\frac{15}{16} > \frac{12}{16}$ so $\frac{15}{16} > \frac{3}{4}$

Exercise 5. Use $<$, $>$, or $=$ to compare these fractions

1. $\frac{3}{4}$ $\frac{9}{16}$

2. $\frac{2}{5}$ $\frac{3}{10}$

3. $\frac{1}{3}$ $\frac{1}{2}$

4. $\frac{10}{16}$ $\frac{5}{8}$

5. $\frac{7}{8}$ $\frac{15}{16}$

Mixed Numbers

A "mixed" number is one that is part whole number and part fraction.

$3\frac{1}{2}$, $4\frac{5}{8}$, $11\frac{2}{3}$ are samples of mixed numbers. Mixed numbers have to be written as fractions only if you're going to multiply or divide them or use them as multipliers or divisors in fraction problems. This change of form is easy to do. Think about $3\frac{1}{2}$. That's 3 whole things and half another. Each of

the 3 wholes has 2 halves ($\frac{2}{2} = 1$). The number 3 is $1+1+1$ or $\frac{2}{2} + \frac{2}{2} + \frac{2}{2}$.

That's $\frac{6}{2}$ and, with the original $\frac{1}{2}$, there's a total of $\frac{7}{2}$. You don't have to think of every one this way; just figure the whole number times the denominator (bottom) and add the numerator (top) $3\frac{1}{2} = \frac{3 \times 2 + 1}{2} = \frac{7}{2}$.

$$3\frac{1}{2} = \frac{3 \times 2 + 1}{2} = \frac{7}{2}$$

$$4\frac{5}{8} = \frac{4 \times 8 + 5}{8} = \frac{32 + 5}{8} = \frac{37}{8}$$

$$2\frac{2}{3} = \frac{2 \times 3 + 2}{3} = \frac{6 + 2}{3} = \frac{8}{3}$$

$$11\frac{5}{9} = \frac{11 \times 9 + 5}{9} = \frac{99 + 5}{9} = \frac{104}{9}$$

Exercise 6. Change these mixed numbers to "top heavy" fractions:

1. $5\frac{7}{8}$

2. $9\frac{2}{3}$

3. $2\frac{1}{2}$

4. $1\frac{1}{8}$

5. $13\frac{1}{2}$

6. $7\frac{3}{4}$

7. $12\frac{2}{5}$

8. $9\frac{5}{9}$

These "top heavy" forms are "work" forms, but they are not usually acceptable answers. If the answer to a calculation comes out a top heavy fraction, it will have to be changed to a mixed number. This can be done by reversing the times and plus to divide and minus. $3\frac{1}{2}$ became $\frac{7}{2}$ by

$\frac{2 \times 3 + 1}{2} \cdot \frac{7}{2}$ can go back to $3\frac{1}{2}$ by dividing 7 and 2.

$$\begin{array}{r} 3\frac{1}{2} \\ 2 \overline{)7} \\ \underline{6} \\ 1 \end{array}$$

The answer is the whole number 3. The remainder 1 is the top number

of the fraction and the divider 2 is the denominator (bottom fraction number).

$$\frac{37}{8} = 8 \overline{)37} = 4\frac{5}{8} \quad \frac{17}{4} = 4 \overline{)17} = 4\frac{1}{4} \quad \frac{35}{3} = 3 \overline{)35} = 11\frac{2}{3}$$

Exercise 7. Reduce these top heavy fractions to mixed numbers.

1. $\frac{27}{8}$ 2. $\frac{13}{5}$ 3. $\frac{93}{8}$ 4. $\frac{66}{7}$ 5. $\frac{25}{2}$

Top heavy fractions may contain common factors as well. They will need to be divided out either before or after the top heavy fraction is changed to a mixed number.

$$\frac{26}{8} = 8 \overline{)26} = 3\frac{2}{8} \quad \text{but } \frac{2}{8} \text{ can be divided by } \frac{2}{2}. \text{ Then } 3\frac{2}{8} \div \frac{2}{2} = 3\frac{1}{4}$$

If you had noticed that both 26 and 8 are even, you could divide out $\frac{2}{2}$ right away and then go for the mixed number. Either way, the mixed number is the same.

$$\frac{26}{8} = \frac{26 \div 2}{8 \div 2} = \frac{13}{4} = 4 \overline{)13} = 3\frac{1}{4}$$

Exercise 8

1. $\frac{65}{10} =$ 2. $\frac{40}{6} =$ 3. $\frac{22}{4} =$ 4. $\frac{22}{8} =$ 5. $\frac{30}{9} =$

Answer Key

Exercise 1	Exercise 2	Exercise 3	Exercise 4
1. $\frac{2}{5}$	1. $\frac{11}{16}$	1. $\frac{3}{4}$	1. $\frac{7}{9}$
2. $\frac{1}{4}$	2. $\frac{9}{20}$	2. $\frac{4}{5}$	2. $\frac{4}{5}$
3. $\frac{11}{12}$	3. $\frac{14}{25}$	3. $\frac{7}{9}$	3. $\frac{1}{2}$
4. $\frac{3}{5}$	4. $\frac{26}{36}$	4. $\frac{4}{5}$	4. $\frac{2}{25}$
5. $\frac{20}{50}$		5. $\frac{1}{2}$	5. $\frac{4}{5}$
6. $\frac{11}{25}$		6. $\frac{2}{3}$	6. $\frac{3}{4}$
7. $\frac{7}{12}$			7. $\frac{8}{9}$
8. $\frac{8}{24}$			8. $\frac{7}{9}$
			9. $\frac{3}{4}$
Exercise 5	Exercise 6	Exercise 7	10. $\frac{12}{17}$
1. $>$	1. $\frac{47}{8}$	1. $3\frac{3}{8}$	11. $\frac{1}{3}$
2. $>$	2. $\frac{29}{3}$	2. $2\frac{3}{5}$	12. $\frac{2}{3}$
3. $<$	3. $\frac{5}{2}$	3. $11\frac{5}{8}$	Exercise 8
4. $=$	4. $\frac{9}{8}$	4. $9\frac{3}{7}$	1. $6\frac{1}{2}$
5. $<$	5. $\frac{27}{2}$	5. $12\frac{1}{2}$	2. $6\frac{2}{3}$
	6. $\frac{31}{4}$		3. $5\frac{1}{2}$
	7. $\frac{62}{5}$		4. $2\frac{3}{4}$
	8. $\frac{86}{9}$		5. $3\frac{1}{3}$

